DETERMINATION OF nth derivative of Algebraic rational Function.

In order to find out the nth derivative of any rational function; we should first find out the partial fractions of that function.

Partial fraction are two forms:

i) One form is that in which the denominator contains linear factors only.

ii) The other form in which the denominator contains linear factor only. as well as quadratic factors.

If the denominator in the partial fraction consists of linear (first degree) expression then its nth derivative can be found by direct application of formula. But the expression in the denominator consist of an second degree, then nth derivative is found by the application of De Moivre's Theorem.

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Case-I : When the denominator of the given expression contains only real linear factors.

Solutions

Q) If $y = \frac{x+3}{(x+1)(x+2)}$, find y_n.

Solution: $y = \frac{x+3}{(x+1)(x+2)}$

$$\frac{x+3}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

$$x+3 = \frac{A(x+2) + B(x+1)}{(x+1)(x+2)}$$

put $x = -2$

$$-2+3 = 0 + B(-2+1)$$

$$1 = -B$$

$\therefore B = -1$

Similarly

$A = 2$

$$\therefore \frac{x+3}{(x+1)(x+2)} = \frac{2}{(x+1)} - \frac{1}{(x+2)}$$

$\because$ we know that, $y = \frac{1}{(ax+b)} = \frac{a^n (-1)^n n!}{(ax+b)^{n+1}}$

$$\therefore y_n = \frac{2 \cdot (-1)^n \cdot n!}{(x+1)^{n+1}} - \frac{1 \cdot (-1)^n n!}{(x+2)^{n+1}}$$

$$= (-1)^n \cdot n! \left[\frac{2}{(x+1)^{n+1}} - \frac{1}{(x+2)^{n+1}}\right] \text{ Ans.}$$

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Q Find y_n, when, $y = \frac{x^n}{(x+1)}$

Solutions: $y = \frac{x^n}{(x+1)}$

Let $(x+1) = t$, so that

$x = t-1$

$\therefore y = \frac{x^n}{(x+1)} = \frac{(t-1)^n}{t}$

$= \frac{t^n - {}^nC_1 t^{n-1} + {}^nC_2 t^{n-2} + \cdots + (-1)^n}{t}$

$= \cancel{(x+1)^n - {}^nC_1 t^{n-1}}$

$= t^{n-1} - {}^nC_1 t^{n-2} + {}^nC_2 t^{n-3} + \cdots + \frac{(-1)^n}{t}$ letting Binomial expansion.

$= (x+1)^{n-1} - {}^nC_1 (x+1)^{n-2} + {}^nC_2 (x+1)^{n-3} + \cdots + \frac{(-1)^n}{(x+1)}$

On ~~taking~~ finding y_n we get

$y_n = \cancel{0+0}$
$0 - 0 + 0 + \cdots + (-1)^n \cdot \frac{n! \, (-1)^n}{(x+1)^{n+1}}$

$= \frac{n!}{(x+1)^{n+1}}$ Ans.